Revisiting the Federated Byzantine Agreement Model

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Example: storing a file reliably in an asynchronous network with 4 servers among which 1 unknown server may fail

- To store the file, make sure at least 3 servers have it
- To retrieve the file, query at least 3 servers



Quorum systems formalize access structures under failure assumptions

We have:

- A set of nodes N
- A quorum system $\mathbb{Q} \subseteq 2^N$ What the nodes access
- A survivor-set system $\mathbb{S} \subseteq 2^N$ At least one survivor set does not fail

 \mathbb{Q} is a quorum system for \mathbb{S} when:

- 1. For liveness: every survivor set includes a quorum
- 2. For safety: every two quorums and one survivor set have nonempty intersection

Every 2 quorums and 1 survivor set must have nonempty intersection



There exists a quorum system for S if and only if every three survivor sets intersect

This is the Q^3 property: $Q^3 \equiv \forall S_1, S_2, S_3 \in \mathbb{S}. S_1 \cap S_2 \cap S_3 \neq \emptyset$



With 3 serves, we cannot tolerate even 1 failure

1 failure = survivor sets of cardinality 2

$$\{n_1, n_2\} \cap \{n_2, n_3\} \cap \{n_3, n_1\} = \emptyset$$

Quorum systems formalize access structures under failure assumptions

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 \mathbb{Q} is a quorum system for \mathbb{S} when:

- 1. For liveness: every survivor set includes a quorum
- 2. For safety: the intersection of any two quorums and a survivor set is nonempty

 \mathbf{Q}^3 : There exists a quorum system for \mathbb{S} if and only if every three survivor sets intersect When \mathbf{Q}^3 holds, we can take $\mathbb{Q} = \mathbb{S}$ the canonical quorum system

Quorum systems are the framework behind the classic distributed-computing toolbox

Reliable broadcast, consensus, shared-memory emulation, group membership, atomic commit, distributed transactional memory, etc. with algorithms such as Bracha broadcast, PBFT, Byzantine Paxos, ...



Great, but developed for centrally managed systems Now we care about permissionless systems

Can traditional quorum systems work in a permissionless system?

- Anybody can unilaterally join or leave the system at any time
- No one knows precisely who is in the system at a given time
- Attackers can try to overwhelm the system with many puppets, also called Sybils
- → A fixed set of quorums will not work



We can use proof-of-stake

- In proof-of-stake, we count money instead of identities
 - E.g. the survivor sets are the sets collectively holding more than 2/3rds of the money
- Caveats
 - Long-range attacks
 - Centralization risk
 - Does wealth reflect trustworthiness or reliability?



Why not let each node make its own failure assumptions and pick its own quorum system?

Each node *n* chooses a survivor set system $S_n \subseteq 2^N$ for itself

 \mathbb{S}_n encodes the assumptions of node n

Two nodes $n \neq n'$ may make different assumptions and have $S_n \neq S_{n'}$ We call this the asymmetric model

Each node *n* chooses a quorum system $\mathbb{Q}_n \subseteq 2^N$ for itself

Requirements:

1. For liveness: every survivor set of n contains a quorum of n

2. For safety:
$$\forall n, n' \colon \forall Q_n \in \mathbb{Q}_n, W_n \in \mathbb{S}_n, Q_{n'} \in \mathbb{Q}_{n'}, W_{n'} \in \mathbb{S}_{n'}.$$

$$Q_n \cap Q_{n'} \cap (W_n \cup W_{n'}) \neq \emptyset$$





There exists a quorum system for $\{\mathbb{S}_n . n \in N\}$ if and only if \mathbf{B}^3 holds

$$S_1, S_2 \in \mathbb{S}_n, S_1', S_2' \in \mathbb{S}_{n'} \Rightarrow S_1 \cap S_1' \cap (S_2 \cup S_2') \neq \emptyset$$

When B^3 holds, we can take $\mathbb{Q}_n = \mathbb{S}_n$ for all n (the canonical quorum system)

We can solve reliable broadcast and shared memory for subsets called guilds

Asymmetric Distributed Trust

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Say nodes are faulty, naïve, or wise

naïve = well-behaved but assumptions violated wise = well-behaved + assumptions satisfied

The set of nodes G is a guild when

1. G is wise and

2. G satisfies it own assumptions

Quorum systems are a key abstraction in distributed fault-tolerant computing for capturing trust at the core of many algorithms for implementing reliable broadcasts. Quorum systems are a key abstraction in distributed fault-tolerant computing for capturing the core of many algorithms for implementing reliable broadcasts. This paper introduces asymmetric Byzanning duorum and a system of the problems. sunptions. They can be found at the core of many algorithms for implementing reliable broadcasts and other problems. This paper introduces asymmetric Bytanine broadcasts is free to choose which combinations of other proshared memory, consensus and other problems. This paper introduces asymmetric Byzanine metric quorum systems that model subjective trust. Every process is free to choose which combinations of other faulty. Asymmetric quorum systems strictly generalize systems that model subjective trust. Every process is free to choose which combinations it considers faulty. Asymmetric quorum systems which have only one global trust assumption for all processes. ^{cesses} it trusts and which ones it considers faulty. Asymmetric quorum systems it considers faulty. This work also presents protocols that implement abstractions of shared memory and broadcast prime. G Sausnes standard Byzantine quorum systems, which have only one global trust assumption for all processes prone to Byzantine faults and asymmetric trust. The model and processes prove to Byzantine faults and asymmetric trust. The model and processes prove to Byzantine faults and asymmetric trust. The model and processes prove to Byzantine faults and asymmetric trust. The model and processes prove to Byzantine faults and asymmetric trust. The model and processes prove to Byzantine faults and asymmetric trust. The model and processes prove to Byzantine faults and asymmetric trust. The model and processes prove to Byzantine faults and asymmetric trust. The model and processes prove to Byzantine faults and asymmetric trust. The model and prove trust as the model as the mod This work also presents protocols that implement abstractions of shared memory and broadcast prime to Byzantine faults and asymmetric trust. The model and broadcast prime algorithms with asymmetric trust.

How do we make sure that \mathbf{B}^3 holds at least for a large fraction of the system?

 \mathbf{B}^3 is an intersection property that must hold for every two nodes

How can it possibly work in open systems where some nodes do not even know each other exist?

Maybe there will be a cartel that everyone trusts to put in their quorums. This seems to be the assumption behind Ripple.

We can do better with FBA!

Federated Byzantine Agreement: make assumptions about assumptions

Each node *n* picks a set of *quorum slices* Sl_n and assumes that it has at least one slice $S \in Sl_n$ such that:

- 1. All members of *S* are well-behaved
- 2. All members of *S* in turn have their assumptions satisfied

W is a minimal survivor-sets/quorum of n when:

- a. $n \in Q$
- b. every member of *W* has a slice in *W*

We will use quorums $\mathbb{Q}_n = \mathbb{S}_n$ { $\mathbb{S}\mathbb{I}_n$. $n \in N$ } determines \mathbb{S}_n and \mathbb{Q}_n for every node n Each node has a unique singleton slice: $SI_i = \{\{i\%4 + 1\}\}$

Every node has the unique quorum: $\{n_1, n_2, n_3, n_4\}$





$\{n, n_a, n_b, n_1, n_2, n_3\} \in \mathbb{Q}_n$









The Internet hypothesis: like the Internet, a global FBA system will be robustly connected

- Nodes make assumptions about failures and about other's assumptions → we can obtain quorum intersection by transitivity
- Hypothesis: market/social forces will keep a global FBA system connected enough to ensure quorum intersection



In FBA, asymmetric quorums are generated collectively

- In the asymmetric model, each nodes picks its survivor sets and quorums
- In FBA, quorums and survivor sets emerge from slices
- The resulting quorum system nevertheless seems to be an asymmetric quorum system
- Algorithms for the asymmetric model should work, but...
 - Quorums are not given upfront, nodes have to compute their quorums

Malicious nodes can forge their slices and lie about them!

- Each node independently chooses its slices
- Quorums depend on the slices of their members
- ➔ Nodes need to know each other's slices

How do they learn each others' slices? By communicating

➔ Malicious nodes can lie about their slices

Without failures, every node has the unique quorum $\{n_1, n_2, n_3, n_4\}$

But, 2 failures compromise quorum intersection!



Without failures, every node has the unique quorum $\{n, n_2, n_3, n_4\}$

But, 2 failures compromise quorum intersection!

Now we have two disjoint quorums: $\{n_1, n_2\}$ and $\{n_3, n_4\}$







In the worst case, malicious nodes make quorums as small as possible



FBA enjoys the quorum-sharing property

"A quorum is a quorum for all its members" We can think of the system as just a set of quorums!

Remember Q is a quorum when:

- a. $n \in Q$
- b. every member of Q has a slice in Q

Also, if Q and Q' are quorums, then so is $Q \cup Q'$



A topology must satisfy 3 axioms

A topology is

- A set of points P (nodes)
- A set of open sets $Open \subseteq 2^P$ (quorums)

With axioms:

- *1.* $\emptyset \in Open$ and $P \in Open$
- 2. If $X \subseteq Open$ then $\bigcup X \in Open$
- 3. If $0, 0' \in Open$ then $0 \cap 0' \in Open$

But, the intersection of two quorums is usually not a quorum!

Semitopology is like topology but without the intersection axiom

A semitopology is

- A set of points *P* (nodes)
- A set of open sets $Open \subseteq 2^P$ (quorums)

With 2 axioms:

- *1.* $\emptyset \in Open$ and $P \in Open$
- 2. If $X \subseteq Open$ then $\bigcup X \in Open$

3. If $0, 0' \in Open$ then $0 \cap 0' \in Open$

We can now turn to familiar topology notions to answer questions about FBA systems

Example: what does it mean to be in agreement?



Recall the definition of continuity at a point p



f is continuous at p when: for every open neighborhood O' of f(p), $f^{-1}(O')$ contains an open neighborhood of p

Take
$$p = 2$$
 and $O' = (3, 4)$

•
$$f(p) = 3.5 \in O'$$

•
$$f^{-1}(O') = (1.5, 2.5)$$
 is open

•
$$2 \in f^{-1}(O')$$

Agreement = continuity

d continuous at n when: for every open neighborhood O' of d(n), $d^{-1}(O')$ contains an open neighborhood of n

Translation: "if n decides v then there is a quorum of n that decides v"



Semitopology: a new topological model of heterogeneous consensus Murdoch J. Gabbay and Giuliano Losa Mar 202 A distributed system is permissionless when participants can join and leave the network without permission from a control normiceion lace in the cones that a control normiceioning outboards. A distributed system is permissionless when participants can join and leave the network without permission protocole some voting evotance and more a without defeat their decign purposes this includes blockchains filecharing protocole some voting evotance and more a without a central permission and a central permission a central permission and a central permission a central permission and a central permission and a central permission a central permission a central permission and a central permission and a central permission and a central permission a central permission and a central permission a central permission a central permission and a central permission a central permission action a central permission a central permission action a central permission action a central permission and a central permission action a central permission action a authority. Many modern distributed systems are naturally permissionless, in the sense that a central permissioning would defeat their design purpose: this includes blockchains, filesharing protocols, some voting systems, and more a narticle onloce natural enchevetame are heterogeneous; narticinante may only have a nartial view of the system and more. By their matter and the system and they may heterogeneous and they may heterogeneous and they may heterogeneous. Would dereat their design purpose: this includes blockchains, filesnaring protocols, some voting systems, and more approximation of concencing permissionless nature, such systems are heterogeneous: participants may only have a partial view of the system, and use may need to generalise it. Jequate, and we may need to generalise it. This is a challenge: how should we understand what heterogeneous consensus is; what mathematical framework might this and how can we use this to build understanding and mathematical models of robust affective and course normicsionlass 29 Ints is a cnauenge: now snoutd we understand what neterogeneous consensus is; what mathematical transmoster require; and how can we use this to build understanding and mathematical models of robust, effective, and secure permissionless every in practice? /stems in practice / In this paper we offer a new definition of heterogeneous consensus, using semitopology as a framework. This is like topology, in twithout the restriction that intercactions of onenc he onen but without the restriction that intersections of opens be open. It without the restriction that intersections of opens be open. Semitopologies have a rich theory which is related to topology, but with its own distinct character and mathematics. 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We give a restriction of semitopologies to witness semitopologies, which algorithmically tractable subclass corresponding to Horn clause theories, having particularly good mathematical properties and study several other basic notions that are specific and novel to semitopologies and etudy how known anantities. algorithmically tractable subclass corresponding to Horn clause theories, having particularly good mathematical properties introduce and study several other basic notions that are specific and novel to semitopologies, and study how known quanties in tonology such as dense subsets and closures display interacting and neefful new behaviour in this new comitopologies is the topologies of topologies of the topologies of topologi Introduce and study several other basic notions that are specific and novel to semitopologies, and study how Known quantities in topology, such as dense subsets and closures, display interesting and useful new behaviour in this new semitopological context. 03.09287₁ Additional Key Words and Phrases: Topology, Semitopology, Permissionless Network. 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37

We discover a zoo of semitopological structures





Fig. 5: Examples of boundary points (Example 6.2.3).

(0)

(2)



0

2

0

Fig. 1: Examples of topens (Example 3.3.3)



2 3



(3

4

3

2

2

0

1

Fig. 10: The semitopologies in Example 10.3.3

Fig. 3: Illustration of Example 4.2.1(3&4)

(2)



0



(a) Regular boundary point of closed neighbourhood that is not intertwined with its interior (Lemma 6.3.11(2))

(b) Regular point in kissing set of closed neighbourhoods, not intertwined with interiors (Lemma 6.3.14(2))

Fig. 7: Two counterexamples



Fig. 8: Example 7.3.5: $|*| \subsetneq *_{\Diamond} \subsetneq \{0, 1, *\}$



0 1

2

(a) A topen that is not strong (Lemma 3.7.2)

0

(b) A transitive set that is not strongly transitive (Lemma 3.7.4(2))



Fig. 9: Illustration of Example 10.1.3(3&4)



A semitopology partitions itself into maximal transitive open sets (topens) plus one non-topen set $Topen(T) \equiv$

- *1. T* is open (is a quorum)
- 2. $0 \land T \land T \land O' \Rightarrow O \land O'$ (T has quorum intersection)

Now show: $Topen(T) \wedge Topen(T') \wedge T \& T' \Rightarrow Topen(T \cup T')$

- 1. By the union axiom, $T \cup T'$ is open
- 2. Consider $O \land T$ and $T' \land O'$ T transitive; O, T' open $\frac{O \land T \quad T \land T'}{O \land T' \quad T' \land O'}$ T' transitive; O, O' opens $\frac{O \land T' \quad T' \land O'}{O \land O'}$

Topens have useful closure properties

Recall, in topology, |R| the closure of R is the set of points whose open neighborhoods all intersect R

If *T* is a topen, we have 1. $\forall 0.0 \in Open \land 0 \land T \Rightarrow T \subseteq |0|$ 2. $\forall R. |R| \land T \Rightarrow R \land T$ Reliable broadcast implements all-or-nothing message broadcasting

There is a designated sender and:

- If n and n' are well-behaved, n delivers message v if and only if n' delivers v
- If the sender is well-behaved, every node eventually delivers its message

Bracha broadcast implements reliable broadcast

The rules:

- 1. announce(sender, v) \Rightarrow vote(n, v)
- 2. $vote(2f + 1, v) \Rightarrow accept(n, v)$
- 3. $accept(f + 1, v) \Rightarrow accept(n, v)$
- 4. $accept(2f + 1, v) \Rightarrow deliver(n, v)$



Bracha broadcast relies on 4 properties

The rules:

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- 4. $accept(2f + 1, v) \Rightarrow deliver(n, v)$

Sufficient properties:

P-1: There are 2f+1 well-behaved nodes

P-2: Every two set of 2f+1 have a well-behaved member in common

P-3: Every set of 2f+1 includes f+1 wellbehaved nodes

P-4: There is one well-behave node among f+1

 $P-3?: \forall O. O \in Open \land O \& T \Rightarrow T \subseteq |O|$ $P-4:? \forall R. |R| \& T \Rightarrow R \& T$

Topological closure generalizes blocking sets

Classic threshold quorum system

- 3f + 1 nodes; f may fail
- quorum threshold is 2f + 1
- blocking set threshold is f + 1

Semitopology

- semitopology with topen *T* that does not fail
- the quorums are the opens
- R blocks n when $n \in |R|$

Bracha broadcast in a semitopology

The rules:

Sufficient properties:

- 1. announce(sender, v) \Rightarrow vote(n, v) P-1: T is an open
- 2. $vote(\mathbf{Q}, \mathbf{v}) \Rightarrow accept(\mathbf{n}, \mathbf{v})$ P-2: T is transitive
- 3. $accept(\mathbf{R}, \mathbf{v}) \land n \in |\mathbf{R}| \Rightarrow accept(\mathbf{n}, \mathbf{v}) P-3: \forall 0.0 \in Open \land 0 \& T \Rightarrow T \subseteq |0|$
- 4. $accept(\boldsymbol{Q}, \boldsymbol{v}) \Rightarrow deliver(\boldsymbol{n}, \boldsymbol{v})$

 $P-4: \forall R. |R| \& T \Rightarrow R \& T$

We can compute closures using a distributed algorithm



We can compute closures using a distributed algorithm

Define $\lim(R) = \bigcup_{i \ge 0} \lim(R)$ where:

- $lim_0(R) = R$
- $lim_{i+1}(R) = lim_i(R) \cup \{n \, . \, \forall S \in \mathbb{Sl}_n . S \notin lim_i(R)\}$ Theorem:

 $|R| = \lim(R)$



Consensus in FBA: quorum certificates do not work

- In algorithms like PBFT, nodes can prove to each other that a quorum *Q* is in a given state by exhibiting a *quorum certificate*, i.e. signed messages from the members of *Q*
- This is not very useful in FBA because the notion of quorum is not shared by everyone
- Solving consensus in FBA is reminiscent of solving consensus in the unauthenticated Byzantine model

Paxos solves consensus in an eventually synchronous crash-stop quorum system

In the consensus problem, nodes start with private inputs and must eventually agree on a common output among the inputs.

Node's outputs are called decisions

Paxos solves consensus in an eventually synchronous crash-stop quorum system

- Nodes execute a sequence of rounds 1,2,3... To simplify, we assume synchronous rounds where each nodes hears from at least a quorum in each round
- Each round has a unique pre-determined leader
- The leader proposes a value and nodes vote for the leader's value
- Any value voted for by a quorum in a given round is decided
- The leader must only propose *safe values*, i.e. values that do not contradict any decision in a previous round

We represent an execution as a table

	Round 1	Round 2	Round 3	Round 4	Round 5
n_1		W	V		V
n_2	V		V		V
n_3					V

A leader proposes the value voted for in the highest round before the current round



Inductively, all previous values are safe for the rounds in which they appear \rightarrow any previous decision must be equal to the value of the highest round

With malicious nodes, we cannot trust leaders or what nodes report

- Nodes and leaders need to double-check that value are safe
- For liveness, a leader must make sure that the value it proposes will be deemed safe by the nodes
- Quorum certificates do not work

Unauthenticated Paxos:

- 4 voting phases per round
- Decision if quorum in the last phase of a round





A value is safe if supported by f + 1 in the previous phase



- Nodes redo the leader's check for themselves
- The leader must not miss a value seen by other nodes, so it uses phase-3 values



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- The leader must not miss a value seen by other nodes, so it uses phase-3 values



We use phases 3 and 4 for the "highest-value" rule



- We use phases 3 and 4 for the "highest-value" rule
- We use phases 1 and 2 to check for safety: a value is safe if supported by f + 1 in the previous phase



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Unauthenticated Byzantine Paxos is like Paxos, but:

There are 4 voting phases per round instead of 1

Leaders use highest phase-3 value and check safety with phase 2

Nodes use highest phase-4 value and check safety with phase 1

Conclusion

The Federated Byzantine Agreement model allows constructing quorum systems in permissionless networks, without proof-of-stake

Quorums in a FBA system are local and form a semitopology, which is a new mathematical object with rich structure and explanatory power

Solving consensus in an FBA system is reminiscent of solving consensus in the unauthenticated BFT model

Open problems:

- Leader election
- Sybil-resistant P2P overlays for FBA
- Cryptography in the FBA model References:
- Semitopology: a new topological model of heterogeneous consensus, arXiv
- Quorum systems in permissionless networks, OPODIS 2022
- Fast and secure global payments with Stellar, SOSP 2019
- Stellar consensus by instantiation, DISC 2019
- The Stellar whitepaper

Non-closure property of leagues in the asymmetric model



Classic $2/3^{rd}$ threshold quorum systems are an instance of FBA

Give every node p the set of slices:

$$\mathbb{S}_p = \{ S \in 2^P : 3|S| = 2|P| \}$$

We obtain a classic BFT quorum system where every node p has the set of survivor-sets/quorums:

$$\mathbb{Q}_p = \{ Q \in 2^P \colon 3|Q| \ge 2|P| \}$$

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Fast and secure global paymeter Fast and secure global paymeter Marta Lokhava, Giuliano Losa*, David Mazières, Gra Marta Lokhava, Giuliano Losa*, David Mazières, Cra Narta Lokhava, Giuliano Losa*, David Mazières, Gra Nicolas Barry, Eli Gafni [†] , Jonathan Jove, Rafal Malinowaki Nicolas Barry, Eli Gafni [†] , Jonathan Jove, Rafal Malinowaki Nicolas Barry, Eli Gafni [†] , Jonathan Jove, Rafal Malinowaki Nicolas Barry, Eli Gafni [†] , Jonathan Jove, Rafal Malinowaki Meridiana Payments are slow and expensive, in part be- baking systems. Stellar is a new global payment network banking systems. Stellar is a new global payment network baking directly transfer digital money anywhere in the ower were key innovation is a secure transaction baking directly transfer digital money anywhere sing a new ow with SCP, each ower and (19), a	aydon Hoare, by, and Jed McCaleb sy, and Jed McCaleb aboring countries. End users pay n uch transfer [32], and a bilateral ar uch transfer [32], and a bilateral ar countries' central banks could or bank cost to \$0.67 per item [2]. Or bank cost to	Chi Univ cachin
Stellar Consensus by Instantiat	ion cient te ncie ats ir	Semitopology:
Giuliano Losa* Eli Gafni Galois, Inc. UCLA giuliano@galois.com eli@ucla.e David Mazières [‡] Stanford http://www.scs.stanford.edu/~dm/ado January 24, 2020	t netw netw netw nobu stell esif ati br y Ke CO2 Ke CO2	Murdoch J. Gabba A distributed system is authority. Many moder would defeat their desi permissionless nature, also have different goo adequate, and we may This is a challenge require; and how can systems in practice?
Abstract Stellar introduced a new type of quorum system called Byzantine Agreement System. A major difference between thi of quorum system and a threshold quorum system is that each has its own, personal notion of a quorum. Thus, unlike in a BFT system, designed for a uniform notion of quorum, even synchrony one well-behaved participant may observe a quor behaved participants, while others may not	a Federated is novel type n participant a traditional in a time of rum of well-	In this paper we of but without the restri- Semitopologies h introduce novel well show how these stru- algorithmically tract introduce and study in topology, such a context.

To task this new problem in a more general setting we abstract

m Systems in Permissionless Networks. versity of Bern Giuliano Losa Stellar Development Foundation n@inf.unibe.ch giuliano@stellar.org Luca Zanolini University of Bern luca.zanolini@unibe.ch November 11, 2029 a new topological model of heterogeneous consensus

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ay and Giuliano Losa

is *permissionless* when participants can join and leave the network without permission from a central ern distributed systems are naturally permissionless, in the sense that a central permissioning authority sign purpose: this includes blockchains, filesharing protocols, some voting systems, and more. By their , such systems are heterogeneous: participants may only have a partial view of the system, and they may pals and beliefs. Thus, the traditional notion of consensus — i.e. system-wide agreement — may not be

e: how should we understand what heterogeneous consensus is; what mathematical framework might this we use this to build understanding and mathematical models of robust, effective, and secure permissionless

offer a new definition of heterogeneous consensus, using *semitopology* as a framework. This is like topology,

have a rich theory which is related to topology, but with its own distinct character and mathematics. We ll-behavedness conditions, including an anti-Hausdorff property and a new notion of 'topen set', and we uctures relate to consensus. We give a restriction of semitopologies to witness semitopologies, which are an ctable subclass corresponding to Horn clause theories, having particularly good mathematical properties. We v several other basic notions that are specific and novel to semitopologies, and study how known quantities as dense subsets and closures, display interesting and useful new behaviour in this new semitopological

Wests and Phrases: Topology, Semitopology, Permissionless Network, Heterogeneous consensus, Horn clause